Analog Simulation of a Hanging 2-Mass, 2-Spring System

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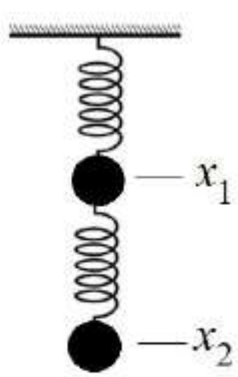
Background:



$$k_1 = k_2 = m_1 = m_2 = 1$$

This system reduces to the following system:

$$\frac{d^2x_1}{dt^2} = -2x_1 + x_2$$

$$\frac{d^2x_2}{dt^2} = x_1 - x_2$$

Positive displacement is down. First and second derivatives of x1 and x2 are their velocity and acceleration, respectively.

Part I. MATLAB Solution

Given some Initial Energy: $x_1(0) = x_2(0) = 1$ and $\dot{x}_1(0) = \dot{x}_2(0) = 0$

Some preliminary work is necessary for Parts I and II:

① $\ddot{x}_1 = -2x_1 + x_2$

② $\ddot{x}_2 = x_1 - x_2$

From ①: $x_2 = \ddot{x}_1 + 2x_1$

Substitute into ②:

$$\frac{d}{dt}\left[\frac{d}{dt}\left(\ddot{x}_1 + 2x_1\right)\right] = x_1 - \left(\ddot{x}_1 + 2x_1\right)$$

$$\frac{d}{dt}\left[\dddot{x}_1 + 2\dot{x}_1\right] = x_1 - \ddot{x}_1 - 2x_1$$

$$\ddddot{x}_1 + 2\ddot{x}_1 = -x_1 - \ddot{x}_1$$

$$\overset{(4\cdot)}{x}_1 + 3\ddot{x}_1 + x_1 = 0$$

or

$$\frac{d^4x_1}{dt^4} + 3\frac{d^2x_1}{dt^2} + x_1 = 0 \quad \text{———} \quad \boxed{1}$$

① $\ddot{x}_1 = -2x_1 + x_2$

② $\ddot{x}_2 = x_1 - x_2$

From ②: $x_1 = \ddot{x}_2 + x_2$

substitute into ①:

$$\frac{d}{dt}\left[\frac{d}{dt}(\dot{x}_2 + x_2)\right] = -2(\ddot{x}_2 + x_2) + x_2$$

$$\frac{d}{dt}\left[\dddot{x}_2 + \dot{x}_2\right] = -2\ddot{x}_2 - 2x_2 + x_2$$

$$\ddddot{x}_2 + \ddot{x}_2 + 2\ddot{x}_2 + x_2 = 0$$

$$x_2^{(4\cdot)} + 3\ddot{x}_2 + x_2 = 0$$

OR

$$\frac{d^4 x_2}{dt^4} + 3\frac{d^2 x_2}{dt^2} + x_2 = 0 \quad \text{—— [2]}$$

Equation [1] and [2] are the same, which means x1 and x2 satisfy the same differential equation. They should have the same general solution, but with different constants depending on the ICs.

Replacing x1 or x2 with just x gives:

$$\frac{d^4x}{dt^4} + 3\frac{d^2x}{dt^2} + x = 0$$

In order to solve this, the following initial conditions are needed for both x1 and x2:

$$x(0),\ \dot{x}(0),\ \ddot{x}(0),\ \dddot{x}(0)$$

The first two are given, so only the second and third derivatives at t=0 remain:

Solving for $\ddot{x}_1(0)$ and $\ddot{x}_2(0)$:

From the Equations (1 and 2) themselves:

$$\ddot{x}_1(0) = -2x_1(0) + x_2(0)$$

$$\ddot{x}_1(0) = -2(1) + 1 = -2 + 1 = -1$$

Or for Part II:

$$-2(a) + a = -a$$

$$\ddot{x}_2(0) = x_1(0) - x_2(0)$$

$$= 1 - 1 = 0$$

or

$$= a - a = 0 \quad \text{(part II)}$$

Solving for $\dddot{x}_1(0)$ and $\dddot{x}_2(0)$:

$$\dddot{x}_1 = -2\dot{x}_1 + \dot{x}_2$$

$$\dddot{x}_1(0) = -2\dot{x}_1(0) + \dot{x}_2(0)$$

$$= -2(0) + 0 = 0$$

and

$$\dddot{x}_2(0) = \dot{x}_1(0) - \dot{x}_2(0) = 0$$

so,

$$\dddot{x}_1(0) = \dddot{x}_2(0) = 0$$

The following conversion to a system of first oder differential equations is necessary for MATLAB work.

(4.)

$$\overset{(4)}{x} + 3\ddot{x} + x = 0$$

$$y_1 = x \quad y_2 = \frac{dx}{dt} \quad y_3 = \frac{d^2x}{dt^2} \quad y_4 = \frac{d^3x}{dt^3}$$

$$\frac{dy_1}{dt} = y_2$$

$$\frac{dy_2}{dt} = y_3$$

$$\frac{dy_3}{dt} = y_4$$

$$\frac{dy_4}{dt} + 3y_3 + y_1 = 0$$

or in matrix form,

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Verification of this system can be done in MATLAB:

```
>> V = odeToVectorField('D4x == -3*D2x - x')

V =

            Y[2]
            Y[3]
            Y[4]
 - Y[1] - 3*Y[3]
```

Where V is a symbolic vector representing the system of first-order differential equations. Each element of the vector is the right side of the set of 1st order equations *Y*[*i*]' = *V*[*i*]. Source: MathWorks

This system requires y1(0), y2(0), y3(0), y4(0)

Y1(0) = x(0) = 1 (x1(0) = x2(0) = 1)

Y2(0) = x'(0) = 0

Y3(0) = x''(0) = -1 or 0 (x1''(0) = -1 but x2''(0) = 0)

Y4(0) = x'''(0) = 0

Therefore, the initial conditions vector for ode45 will be different, depending on whether we are solving for x1 or x2.

For x1: [1 0 -1 0]

But for x2, it will have to be: [1 0 0 0]

I wrote a function for the system that takes the initial conditions for x1 as input, called ic, determines x2 from the equations, and produces the required plots. The initial conditions for x2 are determined from the given second-order equations given ic.

Because the system of 1st order differential equations is the same for both x1 and x2, only one function describing the system of equations is needed.

The code of the main function is below, and the plots produced follow.

```matlab
%{
Hanging 2-mass, 2-spring system m1=m2=k1=k2=1
Takes Initial conditions for m1 as input
Uses ode45 to produce numerical solutions
Kamaljit  S. Chahal, 6/26/15
%}

function mass_spring(ic)

    function dydt = system(~,y)
    dydt = [y(2); y(3); y(4); -3*y(3)- y(1)];
    % The system of 1st order DEs is a 4x1 column vector
    end

options = odeset('AbsTol', 1e-8, 'RelTol', 1e-6);

[T,x1] = ode45(@system, [0 40], ic, options);  % Soln for x1
% [0 40] is t0,tf = the initial and terminal values of t
% ic = initial conditions for mass 1

% Soln for x2
x2(:,1) = x1(:,3)+2*x1(:,1);
x2(:,2) = x1(:,4)+2*x1(:,2);
x2(:,3) = x1(:,1)-x2(:,1);
% ic2 = [1 0 0 0] for x2 are obtained from the above equations and ic1
% It is not necessary to predetermine them and feed them as an input

figure
% Plots x1 vs t
subplot(2,2,1); plot(T, x1(:,1)); title('x1 vs t'); xlabel('t');
ylabel('x1'); axis([0 50 -2 2]);
% Plots x2 vs t
subplot(2,2,2); plot(T, x2(:,1)); title('x2 vs t'); xlabel('t');
ylabel('x2'); axis([0 50 -2 2]);
% Superimposes the previous two plots
subplot(2,2,[3 4]); plot(T, x1(:,1), '--',T, x2(:,1), ':'); title('x1 and x2
vs t'); xlabel('t');
legend('x1', 'x2'); axis([0 50 -2 2]);

figure
% Plots x1' vs x1
subplot(2,2,1); plot(x1(:,1),x1(:,2)); title('dx1/dt vs x1');
xlabel('x1(t)'); ylabel('dx1/dt');
% Plots x2' vs x2
subplot(2,2,2); plot(x2(:,1),x2(:,2)); title('dx2/dt vs x2');
xlabel('x2(t)'); ylabel('dx2/dt');
% Plots x1 vs x2
subplot(2,2,3); plot(x2(:,1),x1(:,1)); title('x1 vs x2'); xlabel('x2');
ylabel('x1'); axis([-1.5 1.5 -1.5 1.5]);
% Plots x1' vs x2'
subplot(2,2,4); plot(x2(:,2),x1(:,2)); title('dx1/dt vs dx2/dt');
xlabel('dx2/dt'); ylabel('dx1/dt');
end
```

```
>> ic1 = [1 0 -1 0]   % Initial conditions for mass 1

ic1 =
    1    0    -1    0

>> mass_spring(ic1)
```

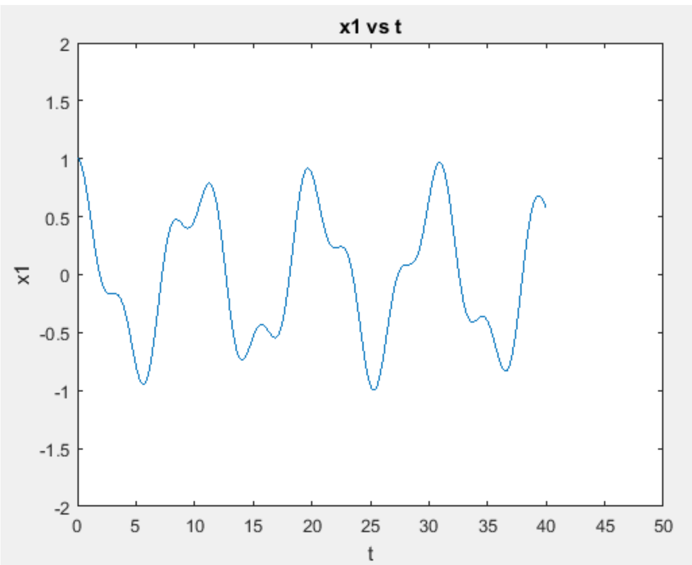


Figure 2   x1 vs t

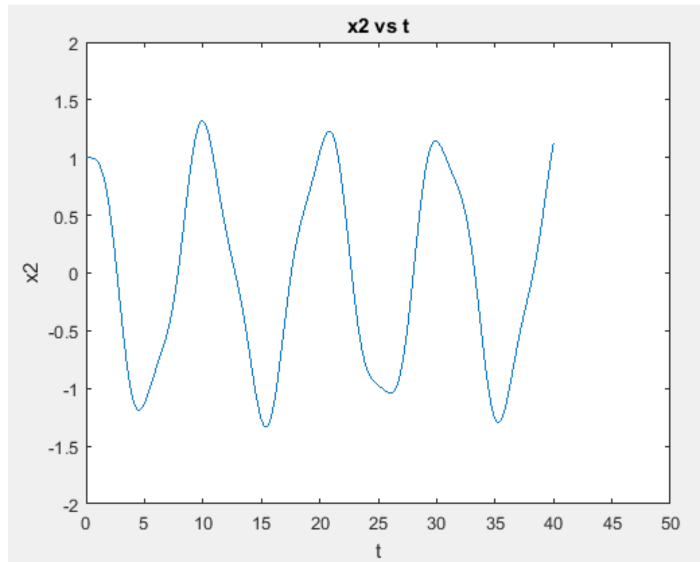


Figure 1   x2 vs t

Superimposing the two shows the masses are sort of synchronized:

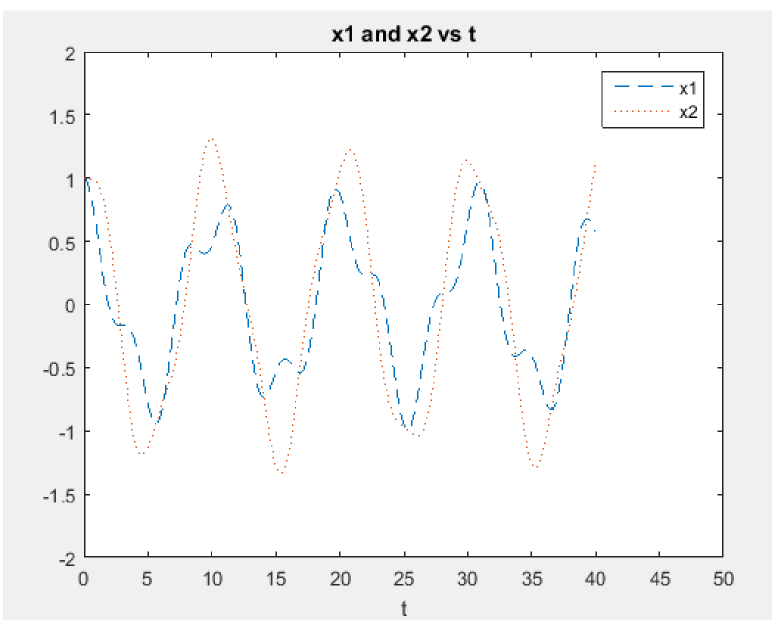


Figure 3   x1 and x2 superimposed

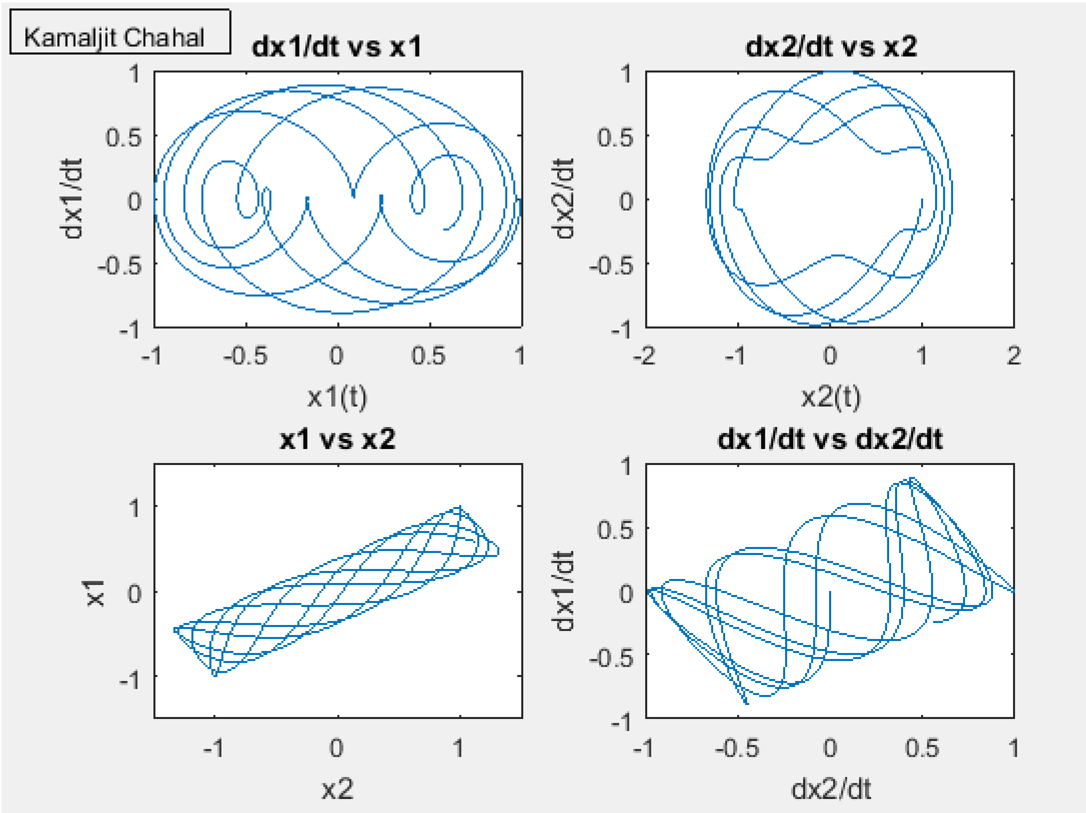


Figure 4: Phase space plots

If we increase tfinal in the line

```
[T,x1] = ode45(@system, [0 400], ic, options);
```

we get cool-looking pseudo-3D plots that show the space filling.

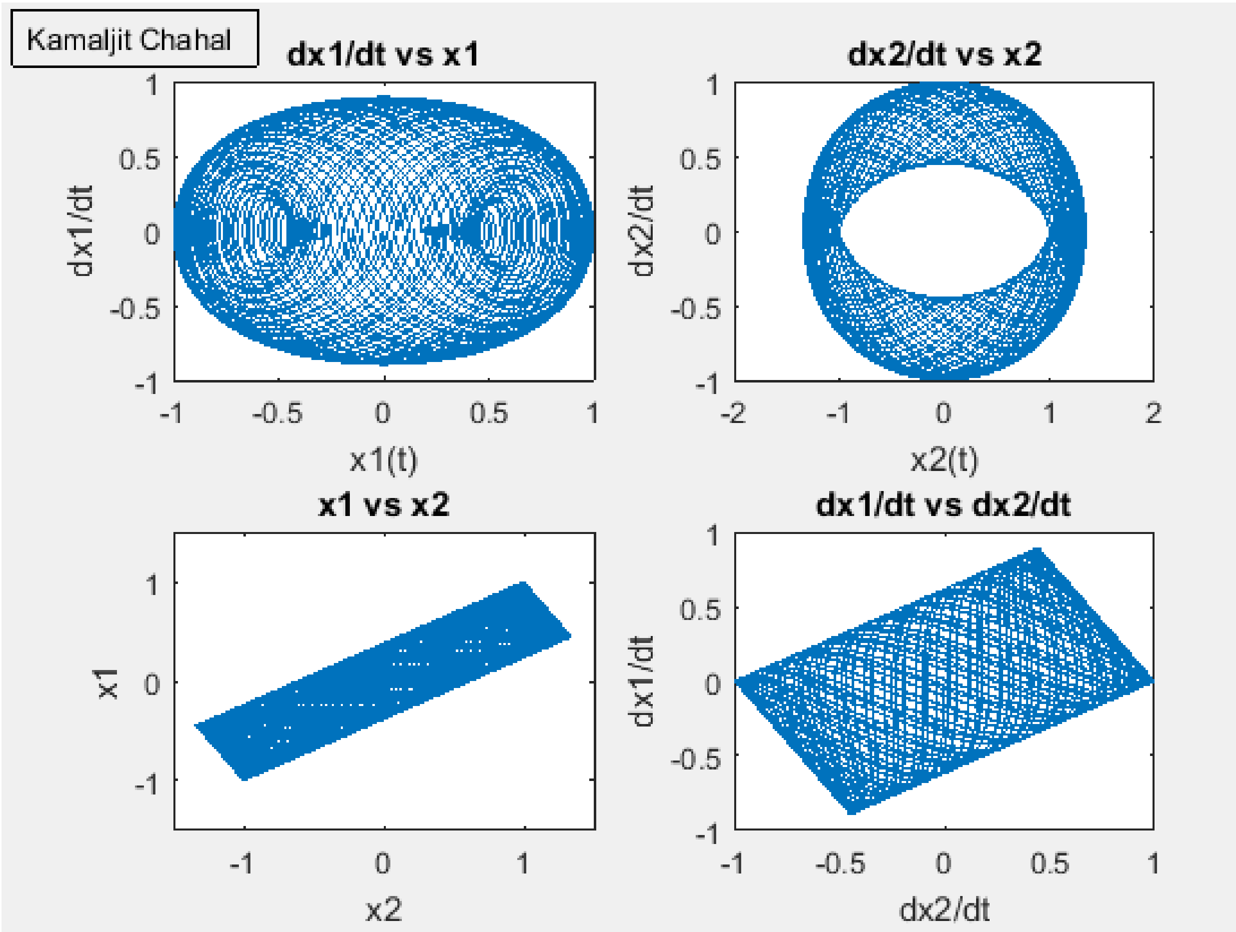


Figure 5: tfinal = 400

Part II. Laplace Solution

This part was the application of the Laplace Transform analysis method to determine the exact analytical solutions for x1(t), x2(t), dx1(t)/dt, and dx2(t)/dt.

It is assumed x1(0) = x2(0) = a while $\dot{x}_1(0) = \dot{x}_2(0) = 0$

See Part I for the determination of the other initial conditions

X1 :

$$\mathcal{L}\left\{ \overset{(4)}{x} + 3\ddot{x} + x = 0 \right\}$$

$$s^4 X(s) - s^3 x(0) - s^2 \dot{x}(0) - s\ddot{x}(0) - \dddot{x}(0)$$

$$+3\left[s^2 X(s) - s x(0) - \dot{x}(0) \right] + X(s) = 0$$

$$x(0) = +a \quad \dot{x}(0) = 0 \quad \ddot{x}(0) = -a \quad \dddot{x}(0) = 0$$

$$(s^4 + 3s^2 + 1) X(s) - s^3 x(0) - \cancel{s^2 \dot{x}(0)} - s\ddot{x}(0) - \cancel{\dddot{x}(0)}$$

$$- 3s x(0) - \cancel{3\dot{x}(0)} = 0$$

$$(s^4 + 3s^2 + 1) X(s) - as^3 + \underbrace{as - 3as}_{-2as} = 0$$

$$X_1(s) = \frac{as^3 + 2as}{s^4 + 3s^2 + 1}$$

X2 : $x(0) = a \quad \dot{x}(0) = 0 \quad \ddot{x}(0) = 0 \quad \dddot{x}(0) = 0$

$$(s^4 + 3s^2 + 1) X_2 - s^3 x(0) - \cancel{s^2 \dot{x}(0)} - \cancel{s\ddot{x}(0)} - \cancel{\dddot{x}(0)}$$

$$- 3s x(0) - \cancel{3\dot{x}(0)} = 0$$

$$X_2(s) = \frac{as^3 + 3as}{s^4 + 3s^2 + 1}$$

I employed an algorithm to find the roots, confirmed by MATLAB, but this still resulted in a tedious dead end.

```
>> p = [1 0 3 0 1];
>> r = roots(p)

r =

  -0.0000 + 1.6180i
  -0.0000 - 1.6180i
  -0.0000 + 0.6180i
  -0.0000 - 0.6180i
```

Because ‘a’ is a constant I pull it out in the following by the scaling property of Laplace.

By completing the square,

$$s^4 + 3s^2 + 1 = \left(s^2 + \frac{3}{2}\right)^2 - \frac{5}{4}$$

$u = s^2$ substitution

$u^2 + 3u + 1 = \left(u + \frac{3}{2}\right)^2 - \frac{5}{4}$

$$\frac{s^3 + 2s}{s^4 + 3s^2 + 1} = \frac{s^3 + 2s}{\left(s^2 + \frac{3}{2}\right)^2 - \frac{5}{4}}$$

Since $(s + a)(s - a) = s^2 - a^2$

the denominator can be rewritten as

$$\left(s^2 + \frac{3}{2} + \sqrt{\frac{5}{4}}\right)\left(s^2 + \frac{3}{2} - \sqrt{\frac{5}{4}}\right)$$

This gives a partial fraction expansion of the form:

$$\frac{s^3 + 2s}{s^4 + 3s^2 + 1} = \frac{As + B}{\left(s^2 + \frac{3}{2} + \sqrt{\frac{5}{4}}\right)} + \frac{Cs + D}{\left(s^2 + \frac{3}{2} - \sqrt{\frac{5}{4}}\right)}$$

$$s^3 + 2s = As\left(s^2 + \frac{3}{2} - \sqrt{\frac{5}{4}}\right) + B\left(s^2 + \frac{3}{2} - \sqrt{\frac{5}{4}}\right)$$

$$+ Cs\left(s^2 + \frac{3}{2} + \sqrt{\frac{5}{4}}\right) + D\left(s^2 + \frac{3}{2} + \sqrt{\frac{5}{4}}\right)$$

$$= As^3 + A\left(\frac{3}{2} - \sqrt{\frac{5}{4}}\right)s + Bs^2 + B\left(\frac{3}{2} - \sqrt{\frac{5}{4}}\right)$$

$$+ Cs^3 + C\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)s + Ds^2 + D\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$$

$$= (A + C)s^3 + \left(A\left(\frac{3}{2} - \sqrt{\frac{5}{4}}\right) + C\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)\right)s$$

$$+ (B + D)s^2 + B\left(\frac{3}{2} - \sqrt{\frac{5}{4}}\right) + D\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$$

Equating coefficients,

$$1 = A + C$$

$$2 = A\left(\frac{3}{2} - \sqrt{\frac{5}{4}}\right) + C\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$$

$$0 = B\left(\frac{3}{2} - \sqrt{\frac{5}{4}}\right) + D\left(\frac{3}{2} + \sqrt{\frac{5}{4}}\right)$$

$$0 = B + D$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ \frac{3}{2} - \sqrt{\frac{5}{4}} & 0 & \frac{3}{2} + \sqrt{\frac{5}{4}} & 0 \\ 0 & \frac{3}{2} - \sqrt{\frac{5}{4}} & 0 & \frac{3}{2} + \sqrt{\frac{5}{4}} \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$A = 0.276$$
$$B = 0$$
$$C = 0.724$$
$$D = 0$$

Therefore, $$\frac{s^3 + 2s}{s^4 + 3s^2 + 1} = \frac{0.276\,s}{\left(s^2 + \frac{3}{2} + \sqrt{\frac{5}{4}}\right)} + \frac{0.724\,s}{\left(s^2 + \frac{3}{2} - \sqrt{\frac{5}{4}}\right)}$$

By the $\mathcal{L}$ pair

$$\mathcal{L}\{\cos \omega t\} \longleftrightarrow \frac{s}{s^2+\omega^2}$$

$\downarrow \mathcal{L}^{-1}$ 1st term in the partial fraction expansion $\qquad \downarrow \mathcal{L}^{-1}$ 2nd term

$$x_1(t) = 0.276\cos\left(\underbrace{\sqrt{\tfrac{3}{2}+\sqrt{\tfrac{5}{4}}}}_{\omega\,=\,\frac{\sqrt{5}}{2}+\frac{1}{2}}\, t\right) + 0.724\cos\left(\underbrace{\sqrt{\tfrac{3}{2}-\sqrt{\tfrac{5}{4}}}}_{\omega\,=\,\frac{\sqrt{5}}{2}-\frac{1}{2}}\, t\right)$$

In the above analytical solution of x1, multiplying by a = 1 results in no change.

MATHCAD work:

$$\left(s^2+\frac{3}{2}-\sqrt{\frac{5}{4}}\right)\cdot\left(s^2+\frac{3}{2}+\sqrt{\frac{5}{4}}\right) \text{ simplify } \rightarrow s^4+3\cdot s^2+1$$

$$A := \begin{pmatrix} 1 & 0 & 1 & 0 \\ \frac{3}{2}-\sqrt{\frac{5}{4}} & 0 & \frac{3}{2}+\sqrt{\frac{5}{4}} & 0 \\ 0 & \frac{3}{2}-\sqrt{\frac{5}{4}} & 0 & \frac{3}{2}+\sqrt{\frac{5}{4}} \\ 0 & 1 & 0 & 1 \end{pmatrix} \qquad y_1 := \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

$$ABCD := A^{-1}\cdot y_1 = \begin{pmatrix} 0.276 \\ 0 \\ 0.724 \\ 0 \end{pmatrix}$$

ω1 :

$$\sqrt{\frac{3}{2}+\sqrt{\frac{5}{4}}} \text{ simplify } \rightarrow \frac{\sqrt{5}}{2}+\frac{1}{2}$$

ω2 :

$$\sqrt{\frac{3}{2}-\sqrt{\frac{5}{4}}} \text{ simplify } \rightarrow \frac{\sqrt{5}}{2}-\frac{1}{2}$$

Taking the inverse Laplace: $x_1(t) := 0.276\cdot\cos\left[\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\cdot t\right] + 0.724\cdot\cos\left[\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\cdot t\right]$

X2:

$$\frac{s^3+3s}{s^4+3s^2+1} = \frac{As+B}{\left(s^2+\frac{3}{2}+\sqrt{\frac{5}{4}}\right)} + \frac{Cs+D}{\left(s^2+\frac{3}{2}-\sqrt{\frac{5}{4}}\right)}$$

$$s^3+3s = As\left(s^2+\frac{3}{2}-\sqrt{\frac{5}{4}}\right) + B\left(s^2+\frac{3}{2}-\sqrt{\frac{5}{4}}\right)$$
$$+ Cs\left(s^2+\frac{3}{2}+\sqrt{\frac{5}{4}}\right) + D\left(s^2+\frac{3}{2}+\sqrt{\frac{5}{4}}\right)$$

The rhs is the same as with X1, ∴,

equating coefficients:

$$1 = A + C$$
$$3 = A\left(\frac{3}{2}-\sqrt{\frac{5}{4}}\right) + C\left(\frac{3}{2}+\sqrt{\frac{5}{4}}\right)$$
$$0 = B\left(\frac{3}{2}-\sqrt{\frac{5}{4}}\right) + D\left(\frac{3}{2}+\sqrt{\frac{5}{4}}\right)$$
$$0 = B + D$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ \frac{3}{2}-\sqrt{\frac{5}{4}} & 0 & \frac{3}{2}+\sqrt{\frac{5}{4}} & 0 \\ 0 & \frac{3}{2}-\sqrt{\frac{5}{4}} & 0 & \frac{3}{2}+\sqrt{\frac{5}{4}} \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

coefficient matrix is the same as with X1

$A = -0.171$
$B = 0$
$C = 1.171$
$D = 0$

see Mathcad file

By the $\mathcal{L}$ pair.

$$\mathcal{L}\{\cos \omega t\} \leftrightarrow \frac{s}{s^2+\omega^2}$$

$X_2(s) =$

$$\frac{s^3+3s}{s^4+3s^2+1} = \frac{-0.171s}{s^2+\underbrace{\frac{3}{2}+\sqrt{\frac{5}{4}}}_{\omega^2}} + \frac{1.171s}{s^2+\underbrace{\frac{3}{2}+\sqrt{\frac{5}{4}}}_{\omega^2}}$$

Taking a = 1

Taking the inverse Laplace of both sides gives

$$X_2(t) = -0.171\cos\left(\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)t\right) + 1.171\cos\left(\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)t\right)$$

MATHCAD work:

For x2:

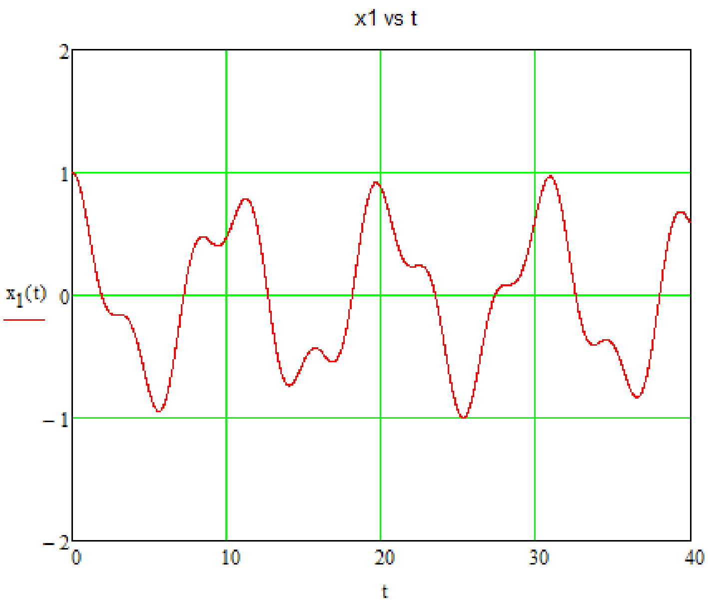
$$A := \begin{pmatrix} 1 & 0 & 1 & 0 \\ \frac{3}{2}-\sqrt{\frac{5}{4}} & 0 & \frac{3}{2}+\sqrt{\frac{5}{4}} & 0 \\ 0 & \frac{3}{2}-\sqrt{\frac{5}{4}} & 0 & \frac{3}{2}+\sqrt{\frac{5}{4}} \\ 0 & 1 & 0 & 1 \end{pmatrix} \qquad y_2 := \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix}$$

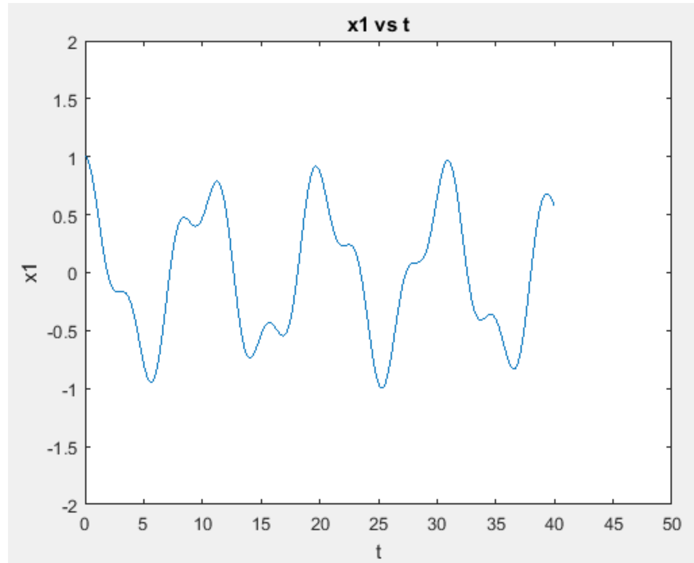
$$ABCD_2 := A^{-1} \cdot y_2 = \begin{pmatrix} -0.171 \\ 0 \\ 1.171 \\ 0 \end{pmatrix}$$

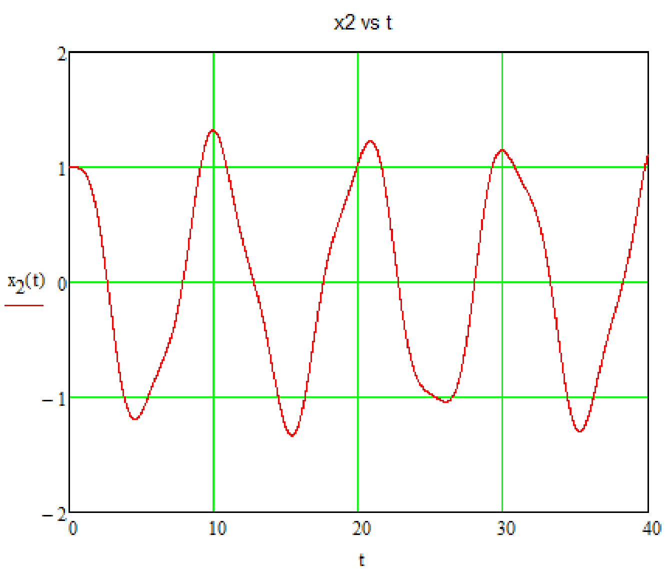
$$x_2(t) := -0.171 \cdot \cos\left[\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\cdot t\right] + 1.171 \cdot \cos\left[\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\cdot t\right]$$

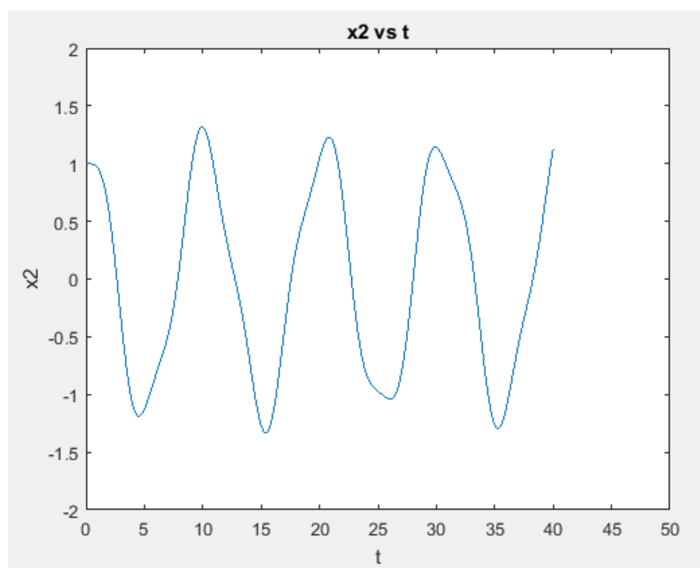
$$t := 0, 0.01 .. 40$$

MATHCAD verification:

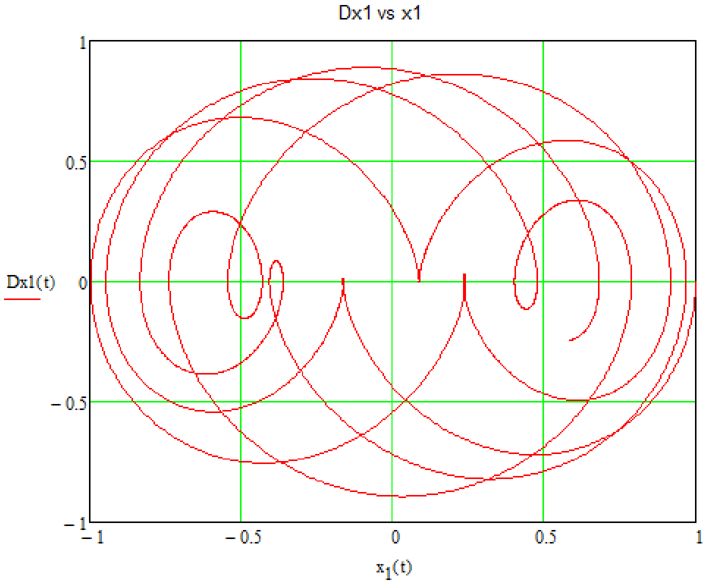




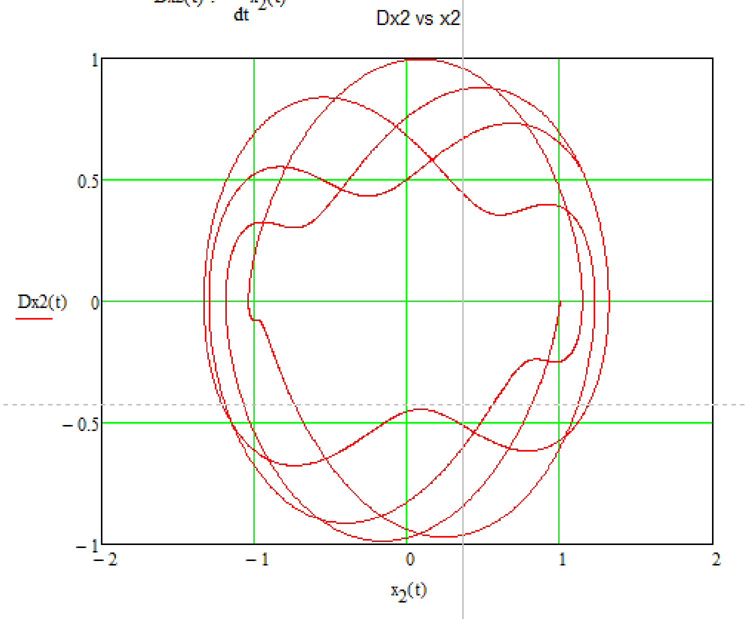


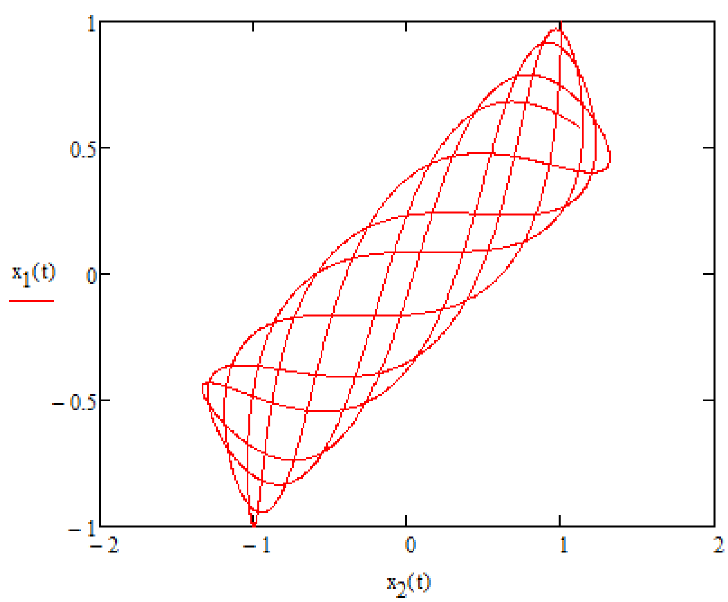


$$Dx1(t) := \frac{d}{dt}x_1(t)$$



$$Dx2(t) := \frac{d}{dt}x_2(t)$$





Periodicity

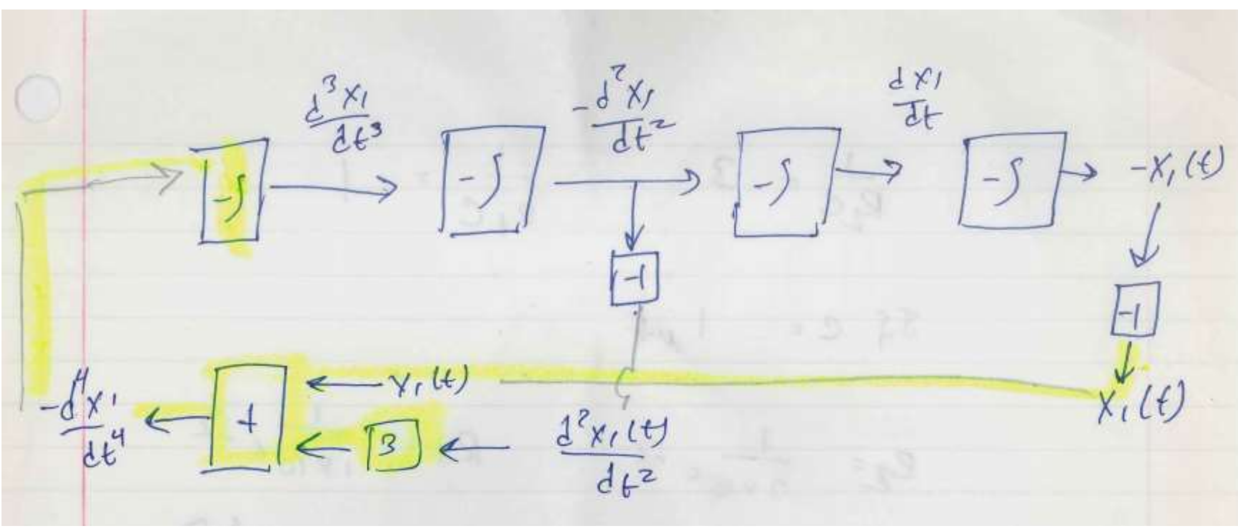
Per lectures 1 and 4, the ratio of the angular frequencies for both x1 and x2 is

$$\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)}{\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)} \text{ simplify } \rightarrow \frac{\sqrt{5}}{2}+\frac{3}{2}$$

And is irrational because of the square root of 5. Therefore, both x1 and x2 are aperiodic. The plots require more than a cursory glance. As was demonstrated in class, the space filling plots never repeat.

Part III. Analog Simulation

Different books seem to present block diagrams in different ways. Here is my block diagram for the 4th order differential equation:



The part highlighted in yellow is implemented through one inverting summing integrator. There are also 3 inverting integrators and 2 inverting amplifiers.

x2 can be obtained by solving the first equation of the given set of 2nd order equations for x2 and employing signals from the above for inputs. It can be implemented using one inverting summing amplifier and one additional inverting amplifier.

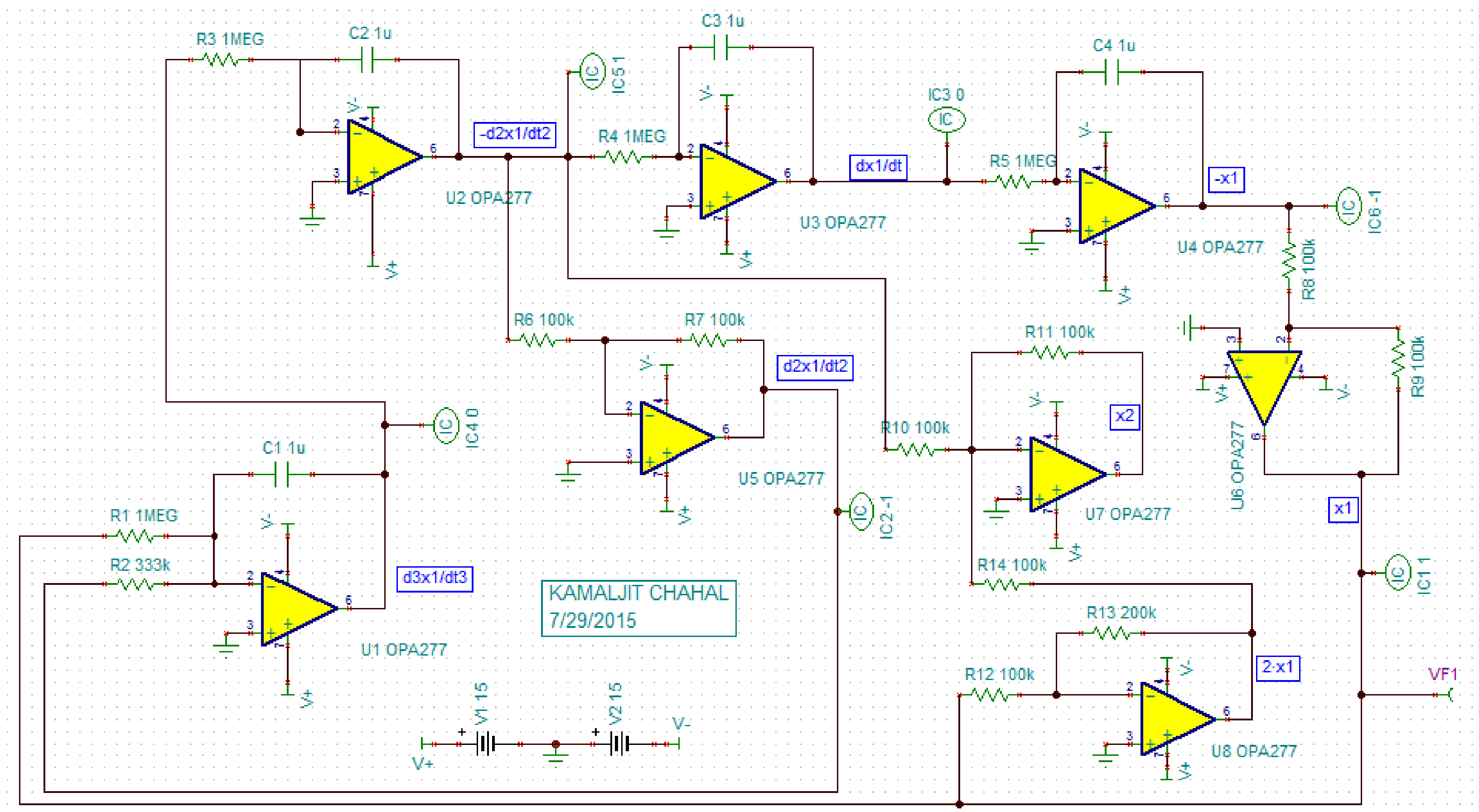
I use Texas Instruments' TINA software (v. 9.3) for virtual circuit simulation because the op amp OPA277 is already part of the components library (It is not for Altium).

Circuit:

Initial conditions are placed for x1 and its derivatives.

See the attached notebook sheet for these calculations.

Here is the circuit diagram:



x1 vs t

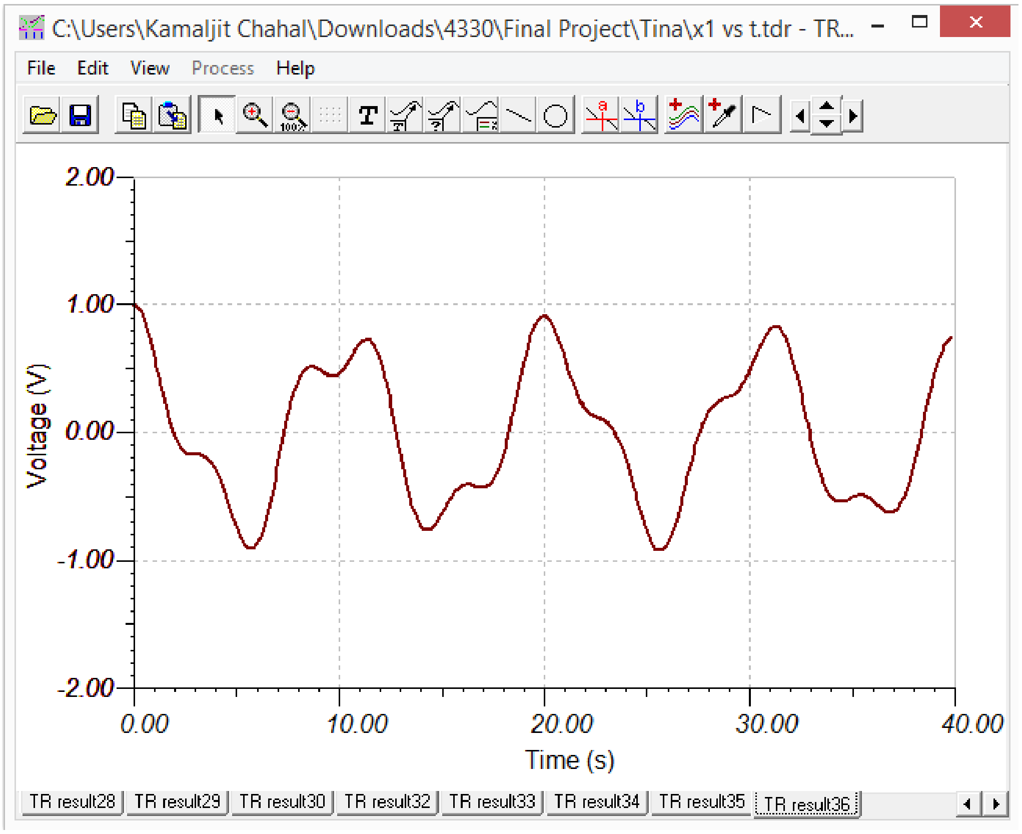


Figure 6: x1 vs t

x2 vs t

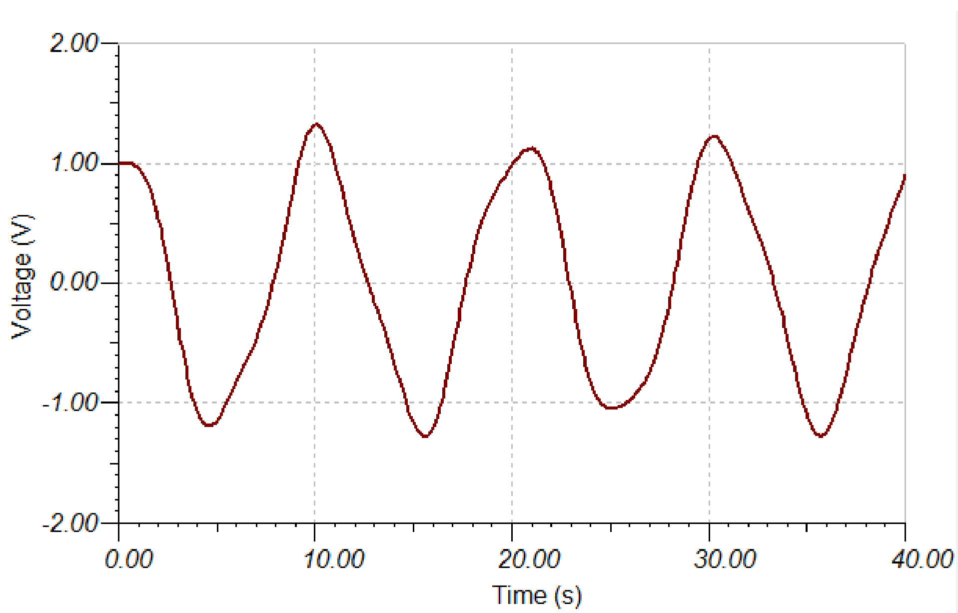


Figure 7: x2 vs t

x1 and x2 vs t

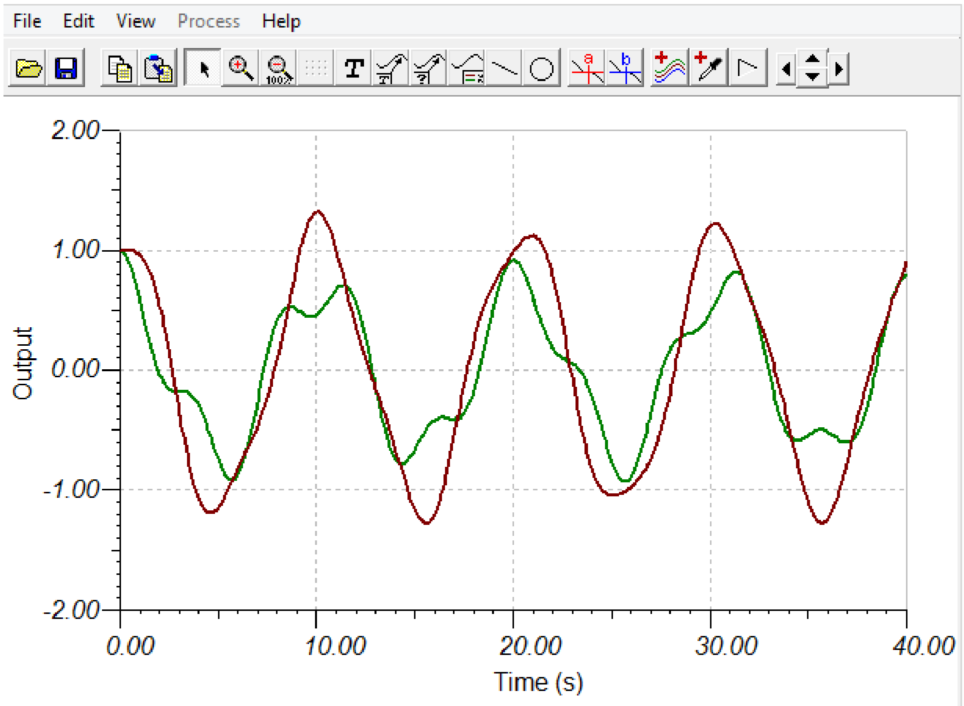
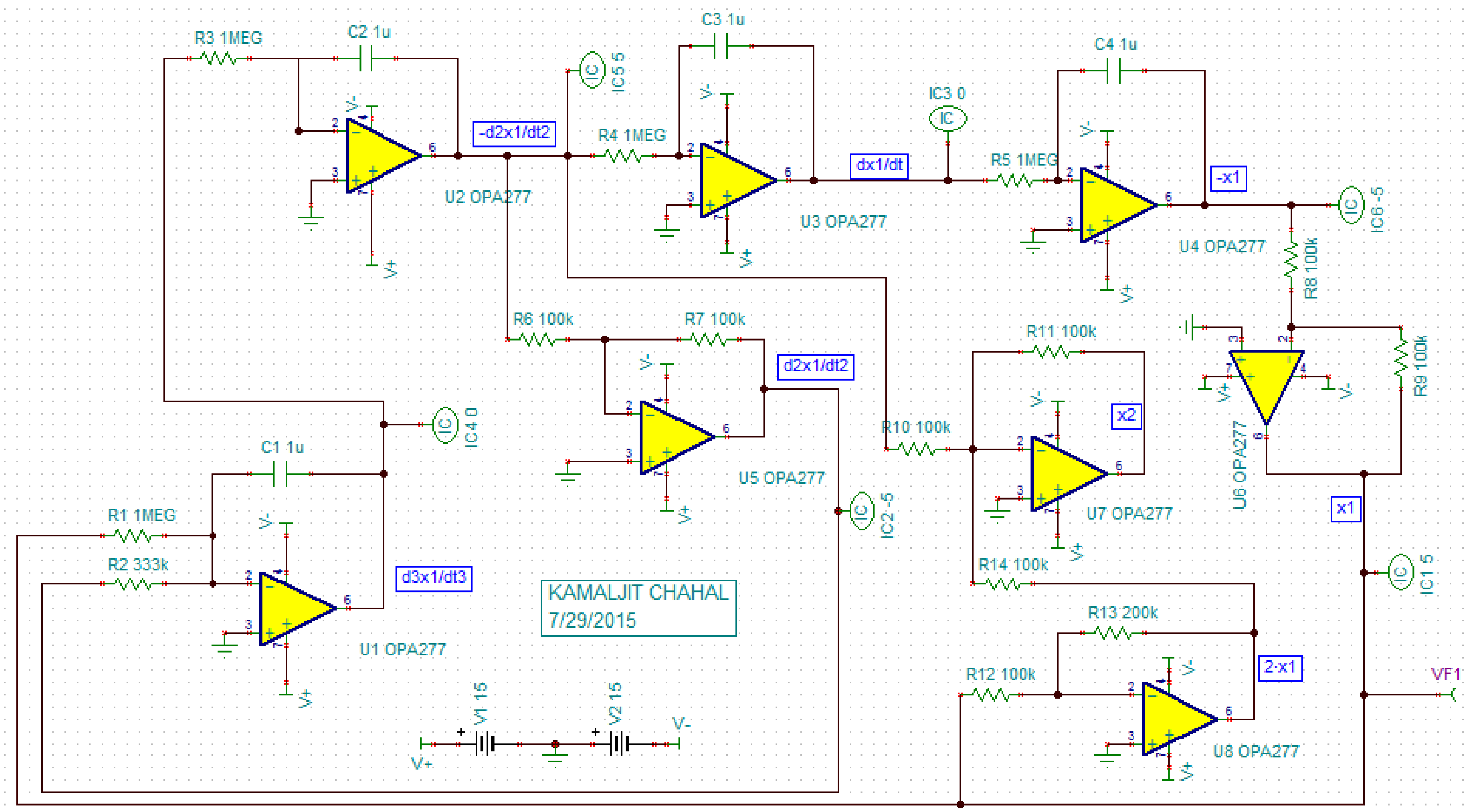


Figure 8: x1 and x2 vs t

I then change the Initial conditions to: x1 {5 , 0 , -5 , 0} ; where the 2nd value is the first derivate at t = 0 and so on; x2 also calculated but not needed.

The initial conditions are accordingly adjusted in the circuit.



The change in the initial conditions scales the magnitude of the signals in question.

x1 and x2 vs t

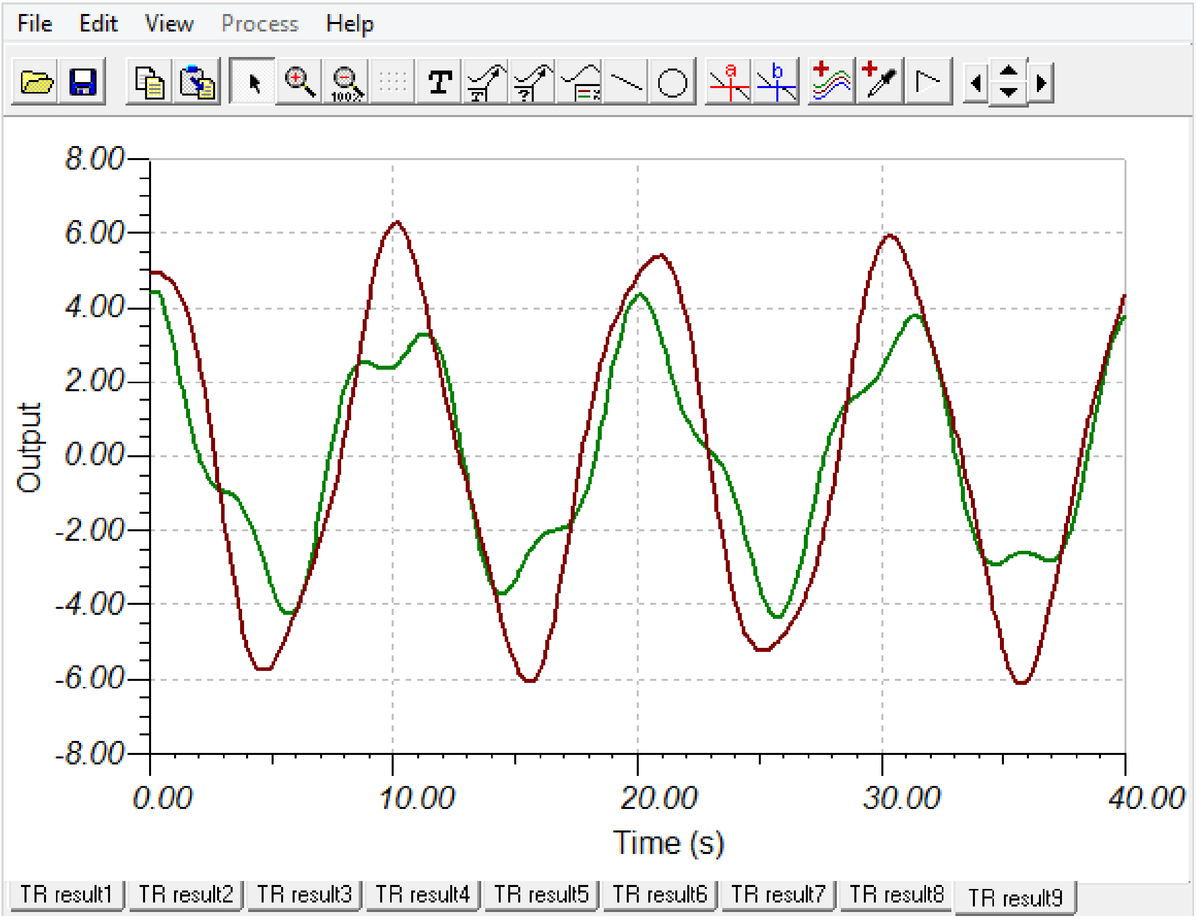


Figure 9: x1 and x2 vs t

Physical implementation

Using OPA277P op amps would have cost close to 50 dollars, not including shipping, so I chose to use the quad version of the OPA277PA instead, the OPA4277PA ($10 each), with double the offset voltage of the OPA277P.

Parametrics

Compare all products in Precision Amplifier

	OPA4277	OPA2277	OPA277
Number of Channels (#)	4	2	1
Total Supply Voltage (Min) (+5V=5, +/-5V=10)	4	4	4
Total Supply Voltage (Max) (+5V=5, +/-5V=10)	36	36	36
Iq per channel (Max) (mA)	0.825	0.825	0.825
Slew Rate (Typ) (V/us)	0.8	0.8	0.8
Vos (Offset Voltage @ 25C) (Max) (mV)	0.05	0.025	0.02
Offset Drift (Typ) (uV/C)	0.15	0.1	0.1

Figure 10: Source: http://www.ti.com/product/opa4277/description

Also, the OPA4277PA does not have offset trim pins.