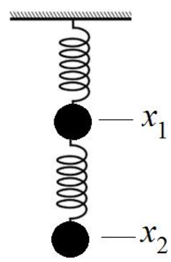
ECE 4330 Bonus Project
(25 points toward the final exam)
Typed report and demonstration due: 3:00 pm on Monday August 3
(Location: Will try to reserve the EE Circuits Lab)

Your report must include a description of the project, simulations, all derivations, circuit diagrams and technical details about design and implementation challenges and their solution

Rules: All work (100%) must be yours to earn credit. If you get any type of help, copy from others or from books or technical papers or online sources, share information/ideas with other students in class, have any similarity between your work (analysis, circuits, simulations, etc.) and somebody else's work, it will automatiically lead to a **penalty of losing 20 points** from your final exam score. Simply put, **DO NOT SUBMIT ANY WORK RELATED TO THIS OPTIONAL PROJECT IF THE WORK IS NOT 100% YOUR OWN**.

You may start on Part (1) right away (refer to the example link provided at the end of the document). Parts (2) you should be able to solve after Lecture 11. Part (3) you can tackle after Lecture 13, but I suggest you start ASAP ordering components (op-amps, capacitors, resistors) after you have read the reference material provided below.

Consider the (ideal) hanging masses system,



where m_1 and m_2 are the upper and lower masses, respectively, and k_1 and k_2 are the upper and lower spring constants, respectively. Letting x_1 and x_2 be the mass displacements from the equilibrium position and applying Newton's second law of motion we may express the system dynamics as,

$$m_1 \frac{d^2 x_1}{dt^2} = -k_1 x_1 - k_2(x_1 - x_2)$$

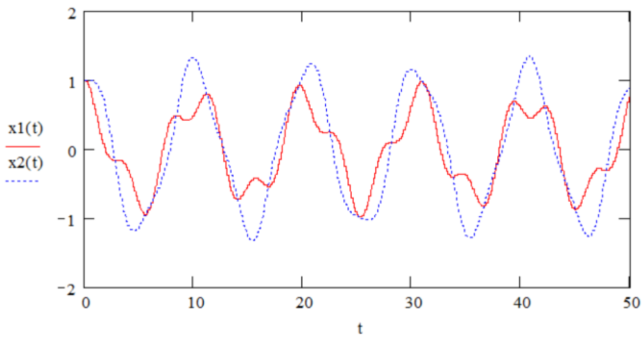
$$m_2 \frac{d^2 x_2}{dt^2} = -k_2(x_2 - x_1)$$

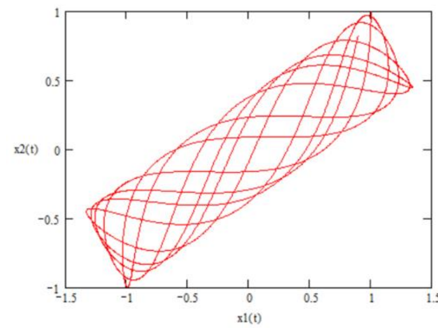
Assuming $k_1 = k_2 = m_1 = m_2 = 1$ leads to the dynamical system,

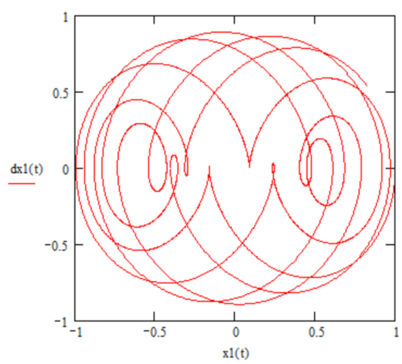
$$\frac{d^2 x_1}{dt^2} = -2x_1 + x_2$$

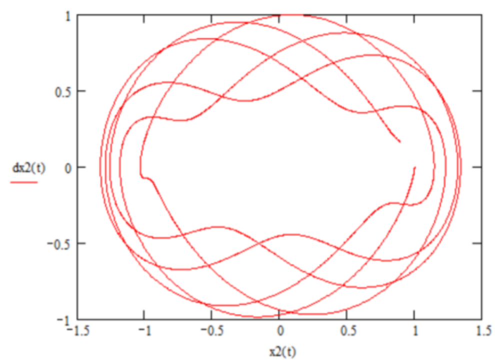
$$\frac{d^2 x_2}{dt^2} = x_1 - x_2$$

(1). [3 points] Employ Matlab and its built-in function ode45 to solve the problem numerically and generate the following plots. Employ initial conditions $x_1(0) = x_2(0) = 1$ and $\dot{x}_1(0) = \dot{x}_2(0) = 0$. (Choose an appropriate upper limit on t for the plots. Note: dx stands for $\dot{x}$)









(2). [7 points]. Assume the initial conditions $x_1(0) = x_2(0) = a$ and $\dot{x}_1(0) = \dot{x}_2(0) = 0$. Apply the Laplace Transform analysis method to obtain the exact analytical solutions for $x_1(t)$, $x_2(t)$, $\dot{x}_1(t)$ *and* $\dot{x}_2(t)$. Show that the solutions are aperiodic. Let $a = 1$ and generate the above plots (from the analytical solutions) using Mathcad.

(3). [15 points] Design and build an analog electronic circuit that solves the system differential equations and generate solutions (equivalent to the above three phase-plane plots) on an oscilloscope screen. Take photos of the circuit and the scope patters and include them in your report. Employ initial conditions $x_1(0) = x_2(0) = 5$ and $\dot{x}_1(0) = \dot{x}_2(0) = 0$.

The following is a resource that you should find helpful for Part (1): Van der Pol Oscillator
The following are resources that you should find helpful for Part (3):

Lecture 13 Laplace Transform

Computing the square-root

Coupled oscillators: Hanging Two mass/spring linear system:

- Physical system
- Solution using analog electronic circuits (analog computer)

Practical integrator circuit: Recommendations: Use Op-amp LF351 or, the more precise OPA277P. Use 1% precision resistors. Employ high-quality capacitors that have low leakage current (Teflon or Polypropylene caps).